DOCUMENT RESUME

ED 464 124 TM 033 805

AUTHOR Fan, Xitao
TITLE Attenuation of Correlation by Measurement Unreliability and Two Approaches for Correcting the Attenuation.
PUB DATE 2002-04-00
NOTE 29p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 1-5, 2002).
PUB TYPE Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Correlation; Measurement Techniques; Monte Carlo Methods; *Reliability; Simulation
IDENTIFIERS Confirmatory Factor Analysis; *Correction for Attenuation

ABSTRACT

This study focused on the issue of measurement reliability and its attenuation on correlation between two composites and two seemingly different approaches for correcting the attenuation. As expected, Monte Carlo simulation results show that correlation coefficients uncorrected for measurement error are systematically biased downward. For the data conditions examined, the two correction approaches provided not only nearly identical and unbiased means, but also near identical confidence intervals for the sampling distribution of the corrected correlation coefficients. The highly comparable results suggest that these two approaches work equally well for these data. It is pointed out that the confirmatory factor analysis modeling approach may be less applicable because of more difficult data conditions at the item level in research practice. The findings point to the importance of reporting measurement reliability information whenever possible. The findings further suggest that correction for attenuation be considered when information about score reliability is available. (Contains 2 tables, 5 figures, and 14 references.) (Author/SLD)

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ED 464 124

Attenuation of Correlation by Measurement Unreliability and Two Approaches for Correcting the Attenuation

Xitao Fan

University of Virginia

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Running Head: Correcting correlation attenuation

Please send correspondence about this paper to:

Xitao Fan
Curry School of Education
University of Virginia
405 Emmet Street South
PO Box 400277
Charlottesville, VA 22904-4277

(434)243-8906
(434)924-1384 (fax)
E-Mail: xfan@virginia.edu
Web: http://www.people.virginia.edu/~xf8d

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TM033805

Paper presented at the 2002 annual meeting of the American Educational Research Association, April 3 (*Session # 30.08*), New Orleans, LA

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Abstract

This study focused on the issue of measurement reliability and its attenuation on correlation between two composites, and two seemingly different approaches for correcting the attenuation. As expected, correlation coefficients uncorrected for measurement error are systematically biased downward. For the data conditions examined, the two correction approaches provided not only near identical and unbiased means , but also near identical confidence intervals for the sampling distribution of the corrected correlation coefficients. The highly comparable results from the two approaches suggest that these two approaches work equally well for these data. It is pointed out that the CFA modeling approach may be less applicable because of more difficult data conditions at the item level in research practice. The findings point to the importance of reporting measurement reliability information whenever possible. The findings further suggest that correction for attenuation should be considered when information about score reliability is available.

Keywords: correlation, reliability, attenuation, confirmatory factor analysis

It is well-known that unreliability in measurement attenuates the statistical relationship between two composites (e.g., Crocker & Algina, 1986; Worthen, White, Fan, & Sudweeks, 1999). Two approaches have been discussed for correcting such attenuation caused by measurement error. The traditional approach is typically discussed within the context of measurement reliability and validity, and sample score reliability coefficients of the two composites of interest are usually used for algebraically correcting the attenuation of correlation cause by measurement unreliability (e.g., Crocker & Algina, 1986; Gulliksen, 1987). The second approach is often discussed within the context of confirmatory factor analysis, or more broadly, structural equation modeling, in which the measurement errors are explicitly modeled, and measurement-error-free correlation between two composites (or factors, latent variables) is thus obtained (Jöreskog & Sörbom, 1989; Loehlin, 1992). It is not clear from the literature how comparable the results from the two seemingly different approaches are. The purpose of this paper is to present the results of an empirical study in which these two approaches for correcting such attenuation on correlation coefficient are systematically compared with each other, and with the correlation coefficients uncorrected such attenuation.

Traditional Approach for Correcting the Attenuation

In classical test theory, the issue of attenuation of correlation between two composites caused by measurement unreliability is usually discussed within the context of score reliability and validity. More specifically, if there are two measured variables X and Y, their correlation is estimated by the Pearson correlation coefficient r_{XY} from a sample. Because the measured variables X and Y contain random measurement error, this correlation coefficient r_{XY} is typically lower than the correlation coefficient between the true scores of the variables T_X and T_Y ($r_{TX,TY}$). As shown by Crocker and Algina (1986, Chapter 10), failure to take into account such

attenuation caused by measurement unreliability may potentially lead to erroneous conclusions about the relationships between the composites, and about measurement validity coefficients. Although $r_{Tx,Ty}$ (correlation between the true scores of X and Y) cannot be obtained directly from measurement data, as is shown in the measurement literature (e.g., Crocker & Algina, 1986; Gulliksen, 1987), the theoretical relationship between $r_{Tx,Ty}$, r_{XY}, and reliability coefficients for composites X and Y (r_{xx}, r_{yy}) is as follows:

$$r_{XY} = r_{Tx,Ty} \sqrt{r_{xx}\ r_{yy}} \qquad (1)$$

So the measurement-error-free relationship between the true scores of X and Y, i.e.; the relationship between X and Y after correcting for the attenuation caused by measurement error, can be expressed as:

$$r_{Tx,Ty} = \frac{r_{XY}}{\sqrt{r_{xx}\ r_{yy}}} \qquad (2)$$

Latent Variable Modeling Approach for Correcting the Attenuation

In confirmatory factor analysis where each latent factor has multiple indicators, measurement error is explicitly modeled in the process. The relationship between two such latent factors can be considered as free from the attenuation caused by the measurement error. An example for two related latent variables (F_X, F_Y), each with four measured indicators (X_1-X_4, Y_1-Y_4), is shown in Figure 1. In this model, r_{FxFy} is considered to represent the true relationship between the two latent variables (F_X, F_Y, respectively) that is not attenuated by the measurement error (modeled as e1 – e8 in Figure 1). This approach for obtaining measurement-error-free

relationship between factors is well-known in the area of structural equation modeling, but is rarely discussed within the context of measurement reliability and validity.

Insert Figure 1 about here

The two seemingly different approaches for correcting the measurement error attenuation of the relationship between two composites are typically discussed in different areas of research or different disciplines. As a result, it is not clear how comparable the results from these two approaches will be. Our literature review indicates that these two approaches have not been compared in empirical studies, so the merits or demerits of each of these two approaches in terms of recovering the true magnitude of the relationship between the two composites (factors) are not clear. This study was designed to shed some light on this issue through systematic comparison of the two approaches. Monte Carlo simulation was used as the tool for this investigation.

Methods

Simulation Design and Data Source

For the Monte Carlo simulation, several potentially relevant aspects were considered: number of items for each composite, magnitude of inter-item correlation within a composite, magnitude of inter-factor correlation (i.e., correlation between the two composites), and sample size. For item number per composite, two conditions were considered: each factor had either four or eight items. For the aspect of inter-item correlation magnitude, five conditions were used (.81, .64, .49, .36, .25), making the inter-item correlations ranging from very high (.81) to relatively low (.25). The aspect of inter-factor correlation had two levels: .4, and .6. Finally, 4 sample size conditions were implemented: 50, 100, 200, 400. The four factors were fully crossed, making the total number of cells to be 80 ($2 \times 5 \times 2 \times 4$). Within each cell, 500 random samples

(replications) were drawn from the specified statistical population, making the total number of replications for the Monte Carlo simulation experiment to be 40,000 [(2 × 5 × 2 × 4) × 500].

Once the inter-item correlation was specified, the population reliability in the form of Cronbach's coefficient alpha could be obtained. Cronbach coefficient alpha takes the form:

$$\alpha = \frac{k}{k-1}(1 - \frac{\sum \sigma_i^2}{\sigma_X^2}) \qquad (3)$$

where k is the number of items within a composite, $\sum \sigma_i^2$ is the sum of item variances, and σ_X^2 is the variance of the composite score. The variance of the composite σ_X^2 is simply the sum of item variances ($\Sigma \sigma_i^2$) and the sum of item covariances ($2\Sigma \sigma_{ij}$):

$$\sigma_X^2 = \sum \sigma_i^2 + 2\sum \sigma_{ij} \qquad (4)$$

In this study, at the item level, normal standardized variables (normally distributed with $\mu = 0$ and $\sigma^2 = 1$) were simulated. The covariance between two standardized variables is simply the correlation between them. So for a composite consisting of k standardized variables with equal inter-item correlation coefficient of ρ, we have the following:

$$\sum \sigma_i^2 = k, \quad \text{and} \quad 2\sum \sigma_{ij} = k(k-1)\rho$$

So, population Cronbach's coefficient alphas for the composites simulated in this study are:

$$\alpha = \frac{k}{k-1}[1 - \frac{k}{k + k(k-1)\rho}] \qquad (5)$$

Table 1 presents the data conditions and the associated population Cronbach's coefficient

alphas for the composites simulated. Because inter-factor correlation (0.4 and 0.6 respectively in this study) does not affect the reliability of the composites, the population Cronbach's coefficient alphas were the same for inter-factor correlation of 0.4 and 0.6. The population Cronbach's coefficient α presented in Table 1 shows that score reliability conditions examined in this study ranged from marginal reliability ($\alpha = 0.57$) to very high reliability (($\alpha = 0.97$).

Insert Table 1 about here

The simulation is carried out by using the SAS system, and a combination of SAS/MACRO, SAS/BASE, SAS/PROC IML (Interactive Matrix Language), and SAS/STAT are used for accomplishing the tasks. Confirmatory factor analysis (CFA) was implemented by using SAS/PROC CALIS (Covariance Analysis of LInear Structures). Random normal variables were generated by using the random normal number generator (RANNOR) in SAS. The population inter-variable correlations was obtained from the two-factor model in Figure 1 based on the following (Jöreskog & Sörbom, 1989):

$$\Sigma = \Lambda \Phi \Lambda' + \Theta \qquad (6)$$

where, Σ is the population covariance matrix (correlation matrix for our standardized variables), Λ is the matrix of population pattern coefficients in Figure 1, Φ is the population correlation matrix for the two factors, and Θ is the covariance matrix of population residuals for the items. For simulating the specified population inter-variable correlations in Σ in Equation 6, the matrix decomposition procedures (see Kaiser & Dickman, 1962) were implemented.

Results and Discussions

Preliminary Results

In fitting a confirmatory factor analysis model to a sample data, we may sometimes encounter the problem of non-convergence, i.e., we may fail to obtain model parameter estimates due to non-convergence of a sample. This possibility was checked. The results show that, for the simple two-factor CFA model shown in Figure 1, all samples converged, with small samples requiring more iterations for achieving convergence than larger samples, consistent with what was shown in the literature (e.g., Fan & Wang, 1998) . Table 2 presents the average number of iterations required for achieving convergence for different sample size conditions.

Insert Table 2 about here

Results for 4-Item Composites

Figure 2 graphically presents the results for correlation coefficients between two composites, each consisting of four items. In Figure 2, the population correlation between the two factors is 0.40. The 95% confidence intervals (upper and lower limits) for three types of correlation coefficients between two composites are displayed, with the line in the middle of the bar indicating the mean of the correlations from 500 samples. These are empirical confidence intervals based on exact percentile points, not on standard errors. As a result, the construction of these confidence intervals does not assume normal distribution. The three types of correlations displayed are: a) the correlation coefficient of two composites without correcting for the attenuation of measurement unreliability (R_xy), b) correlation coefficient between two composites corrected for the attenuation of measurement unreliability (R_xy Corrected; algebraic correction based on Equation 2), and c) the measurement-error-free correlation between two

latent factors as in Figure 1 (R_CFA). The dashed horizontal line at 0.4 represents the population correlation between the two latent variables.

Insert Figure 2 about here

Several phenomena stand out in Figure 2. First, the correlation coefficient between two composites uncorrected for measurement error has obvious systematic downward bias, as indicated by the fact that the mean of the uncorrected correlation coefficients is systematically lower than the population correlation of 0.4. The more measurement error the composites contain (i.e., the lower the reliability Rxx), the more downward bias the correlation has. When population correlation is 0.40, the average uncorrected correlation coefficients are 0.23 (for α=0.57), 0.27 (for α=0.69), 0.31 (for α=0.79), 0.34 (for α=0.88), and 0.38 (for α=0.94), respectively. By itself, this is not surprising, because this fact of downward bias is well known. What is surprising is the observation that the downward bias can be such that even the upper limit of the 95% confidence interval for the uncorrected correlation may still be lower than the population correlation, especially when the sample size is relatively large (e.g., N = 200, 400) and measurement reliability is relatively low (e.g., Rxx = .57, .69).

Second, for the uncorrected correlation coefficient (R_xy), the confidence interval width (i.e., sampling variation) does not change with measurement reliability. The confidence interval widths of the other two correlation coefficients corrected for measurement error (R_xy Corrected, R_CFA), however, are related to measurement reliability: the lower the measurement reliability, the more sampling variation for these two types correlation coefficients.

Third, the two approaches for correcting the attenuation of measurement error on correlation coefficient (algebraic correction based on Equation 2, and that based on confirmatory

factor analysis) worked remarkably well in capturing the population correlation, because the means of the corrected correlation coefficients based on these two approaches are right on the target of the population correlation of 0.40, for all the five conditions of measurement reliability considered in this study. This shows that these two correction approaches produced unbiased sample estimates.

Fourth, when sample size was small (e.g., N = 50), some slight differences occurred in the confidence interval widths based on the two correction approaches. But the results from the two correction approaches generally showed highly consistent results, both in terms of their almost identical means, and in terms of their almost identical confidence interval widths (upper and lower limits).

Figure 3 displays the confidence intervals of the three types of correlation coefficients between two composites when the population correlation between the two is 0.60. Here, the results essentially replicate those in Figure 2. The downward bias of the uncorrected correlation coefficients between two composites, however, appears to be more severe. When population correlation is 0.60, the means of the sample correlation coefficients uncorrected for the attenuation caused by measurement error are 0.34 (for α=0.57), 0.41(for α=0.69), 0.47 (for α=0.79), 0.52 (for α=0.88), and 0.56 (for α=0.94), respectively. When measurement reliability is relatively low (e.g., α = 0.57, 0.69), even the confidence interval's upper limit of the sample correlations may be quite a bit lower than the population correlation of 0.60, let alone the mean of the uncorrected sample correlations. In some situations, the upper limit of the uncorrected sample correlation confidence interval does not even overlap with the lower confidence limit of the corrected sample correlations (e.g., N = 400, α = 0.57, 0.69). This suggests that the attenuation on sample correlation efficient caused by measurement error may be more severe than many

researchers realize. In many research situations, it is not uncommon to have measurement reliability in the range of 0.60-0.80. Under such conditions, even the upper confidence interval limit itself may fail to capture the true correlation between two composites.

Insert Figure 3 about here

Figures 4 and 5 present the findings for correlations between two 8-item composites, for inter-factor correlation of 0.4 (Figure 4) and 0.6 (Figure 5) respectively. The general observations in these two figures are closely comparable to those discussed above for the situation of 4-item composites, thus not repeated here.

Insert Figures 4 and 5 about here

The findings in this study suggest several things related to our research practice. First, it is important to report measurement reliability in a research study, even if the study does not focus on measurement issues. As discussed by Wilkinson and The APA Task Force on Statistical Inference (1999), " . . . authors should provide reliability coefficients of the scores for the data being analyzed even when the focus of their research is not psychometric. Interpreting the size of the observed effects requires an assessment of the reliability of the scores" (p.596). It is obvious from the figures presented above that reporting measurement reliability helps readers to evaluate the magnitudes of correlation coefficients reported in a study.

Although reporting measurement reliability appears to be a simple task that makes common sense in research, this is still far from being actual common research practice. Two decades ago, in a review of the articles in the American Educational Research Journal (AERJ),

Willson (1980) reported that less than 50% of the published studies did not report measurement reliability, and only about 37% of the studies reported score reliability coefficients of their own data. Willson commented that ". . . reliability is unreported in almost half the published research is . . . inexcusable at this late date" (pp. 8-9). More recently, Yin and Fan (2000) reported that only about a dismal 8% of the published studies involving the use of the Beck Depression Inventory actually reported reliability coefficients for their own data. The fact that only such a small percentage of studies reported their measurement reliabilities ". . . shows that the concept of test score reliability has not generally prevailed . . . and research practice . . . still leaves much to be desired" (Yin & Fan, 2000, p. 210).

Reporting measurement reliability for data used in analyses is more than a psychometric concern, however. As clearly shown in Figures 2 to 5 in this study, measurement reliability is directly related to our interpretation of statistical analysis results. As Thompson (1994) discussed, "the failure to consider score reliability in substantive research may exact a toll on the interpretations within research studies. For example, we may conduct studies that could not possibly yield noteworthy effect sizes given that score reliability inherently attenuates effect sizes. Or we may not accurately interpret the effect sizes in our studies if we do not consider the reliability of the scores we are actually analyzing." (p. 840).

Second, since both correction approaches work so well in providing unbiased estimates for correlation between composite measures, it appears reasonable that correction for correlation attenuation caused by measurement error should be done whenever measurement reliability coefficients are available. In applied research practice in education and psychology, it is not uncommon to have somewhat low or moderate measurement reliability (e.g., in the range of 0.60-0.80). As shown in several meta-analytic reliability generalization studies (e.g., Capraro, Capraro,

& Henson, 2001; Caruso, 2000; Yin & Fan, 2000), measurement reliabilities in psychological studies were often as low or lower than 0.60-0.80. As shown in Figures 2 to 5, when measurement reliability is in this range or lower, the downward bias of the uncorrected correlation between two composite measures could be substantial. It is possible that such attenuation of correlation caused by measurement error may have camouflaged many meaningful relationships from researchers.

It should be pointed out that, although the two approaches for correcting attenuation caused by measurement unreliability produced highly comparable results, this does not mean that both approaches are readily applicable in all situations in which there are composites with multiple items. In general, it is typically difficult to model item level data in practice. Gorsuch (1997, p. 315-316) discussed several reasons for the difficulty of modeling item level data in practice: a) items often have low reliability, b) item often contain confounding variance in addition to the construct being measured, (c) item distributions often differ from each other, d) item scores are typically a set of ordered categories rather than continuous. For this reason, the approach of algebraic correction for attenuation based on Equation 2 is more readily usable in research practice. In this sense, these two approaches may not be comparable in actual application, despite their comparable results for the ideal data conditions considered in this study.

Summary and Conclusions

This study focused on the issue of measurement reliability and its attenuation on correlation between two composites. The simulation design allowed systematic comparison of the traditional approach for algebraically correcting for attenuation of relationship caused by measurement error, and the modeling approach based on confirmatory factor analysis in which measurement error is specifically modeled. The study provides useful information for better

understanding of the issue of attenuation caused by measurement error, and about two different approaches for correcting such attenuation. Four factors were considered in this study: composite reliability, number of items comprising the composites, inter-factor correlation, and sample size. Within each cell condition, 500 replications were conducted to estimate the sampling distributions of the uncorrected and corrected correlation coefficients. The findings show that, as expected, correlation coefficients uncorrected for measurement error are systematically biased downward. The magnitude of such downward bias is related to measurement reliability of the composite in reverse direction: the lower the reliability, the larger the magnitude of the downward bias. When measurement reliability is low or moderate (e.g., 0.60-0.80), not only the average of such sample correlations may be substantially lower than the population parameter, but even the upper confidence interval limit of the uncorrected sample correlations may fail to capture the population correlation.

For the data conditions considered, the two correction approaches provided not only near identical and unbiased means , but also near identical confidence intervals for the sampling distribution of the corrected correlation coefficients. The highly comparable results from the two correction approaches suggest that these two approaches work equally well for these data. It is pointed out, however, that the CFA modeling approach may be less applicable in research practice due to more difficult data conditions at the item level in research practice. The findings in this study point to the importance of reporting measurement reliability information in research practice whenever possible. The findings further suggest that correction for attenuation should be considered when information about score reliability is available.

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Table 1 Data Conditions and Reliability for the Composites

(Inter-Factor Correlation is 0.4 and 0.6)

Number of Items in Composite	Inter-Item Correlation	Composite Reliability (α)
4	0.25	0.5714
	0.36	0.6923
	0.49	0.7935
	0.64	0.8767
	0.81	0.9446
8	0.25	0.7273
	0.36	0.8182
	0.49	0.8849
	0.64	0.9343
	0.81	0.9715

Table 2 Number of Iterations for Achieving Convergence in CFA Modeling

	N	Average	Minimum	Maximum
4-Item Composites				
	50	12.78	4	50
	100	8.61	4	50
	200	6.53	3	33
	400	5.22	3	15
8-Item Composites				
	50	9.71	5	50
	100	7.09	5	50
	200	5.77	4	11
	400	4.82	4	8

Figure Captions

Figure 1 A Correlated Two Factor Model with Four Indicators for Each Factor

Figure 2 95% CI for Correlations between 4-Item Composites: Uncorrected (R_xy), Corrected for Unreliability (R_xy Corrected), and that from CFA Model (R_CFA) - Inter-Factor $\rho = 0.40$

Figure 3 95% CI for Correlations between 4-Item Composites: Uncorrected (R_xy), Corrected for Unreliability (R_xy Corrected), and that from CFA Model (R_CFA) - Inter-Factor $\rho = 0.60$

Figure 4 95% CI for Correlations between 8-Item Composites: Uncorrected (R_xy), Corrected for Unreliability (R_xy Corrected), and that from CFA Model (R_CFA) - Inter-Factor $\rho = 0.40$

Figure 5 95% CI for Correlations between 8-Item Composites: Uncorrected (R_xy), Corrected for Unreliability (R_xy Corrected), and that from CFA Model (R_CFA) - Inter-Factor $\rho = 0.60$

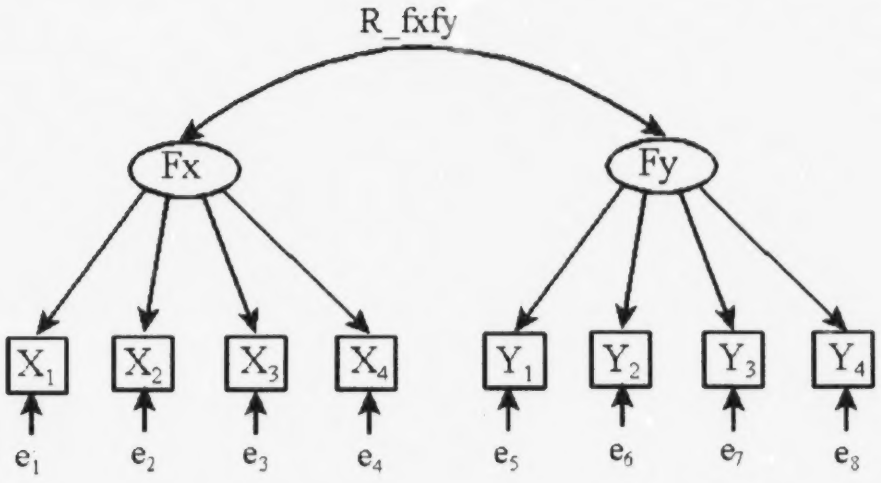

Figure 1 A Correlated Two Factor Model with Four Indicators for Each Factor

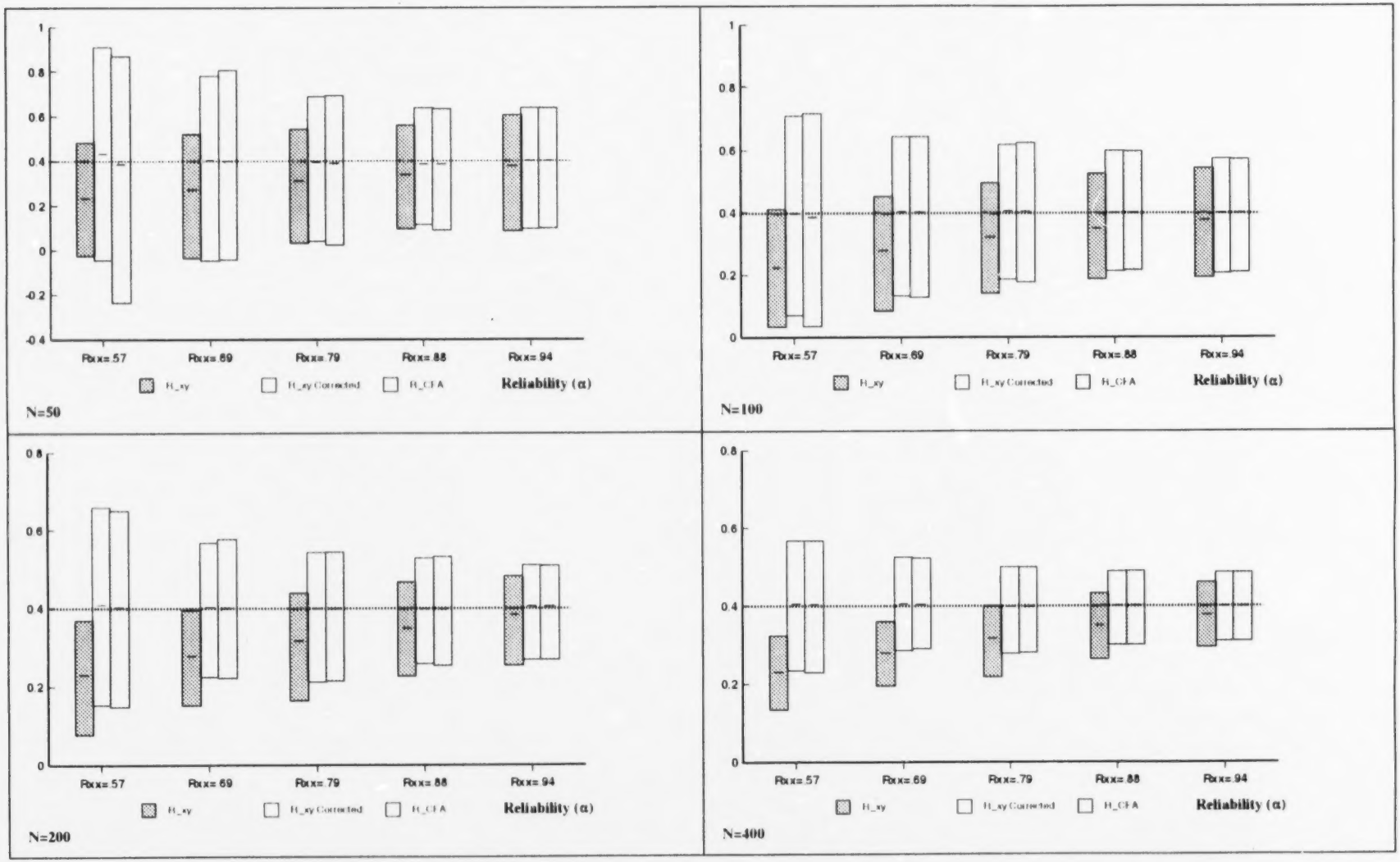

Figure 2 95% CI for Correlations between 4-Item Composites: Uncorrected (R_xy), Corrected for Unreliability (R_xy Corrected), and that from CFA Model (R_CFA) - Inter-Factor $\rho = 0.40$

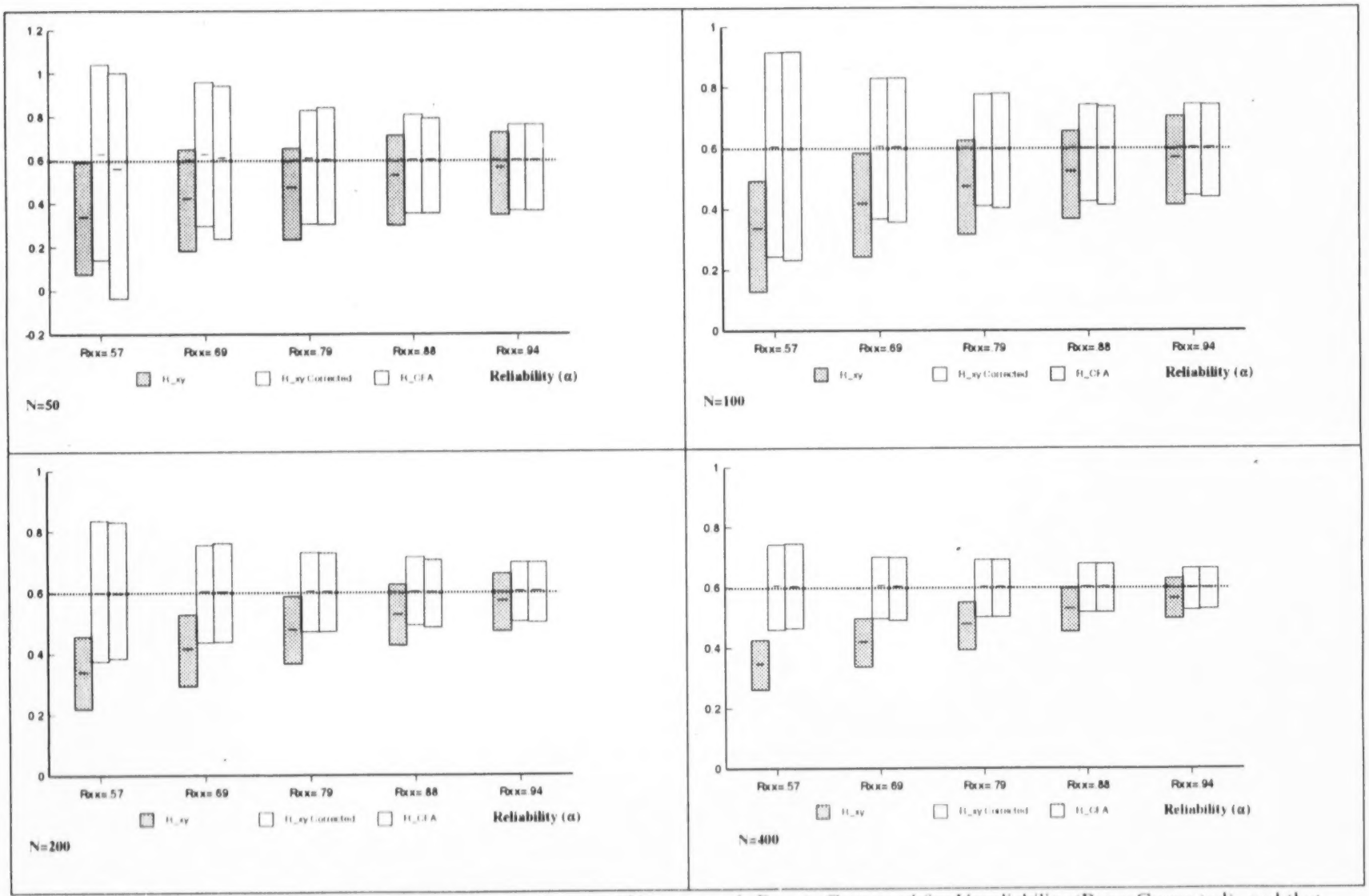

Figure 3 95% CI for Correlations between 4-Item Composites: Uncorrected (R_xy), Corrected for Unreliability (R_xy Corrected), and that from CFA Model (R_CFA) - Inter-Factor ρ = 0.60

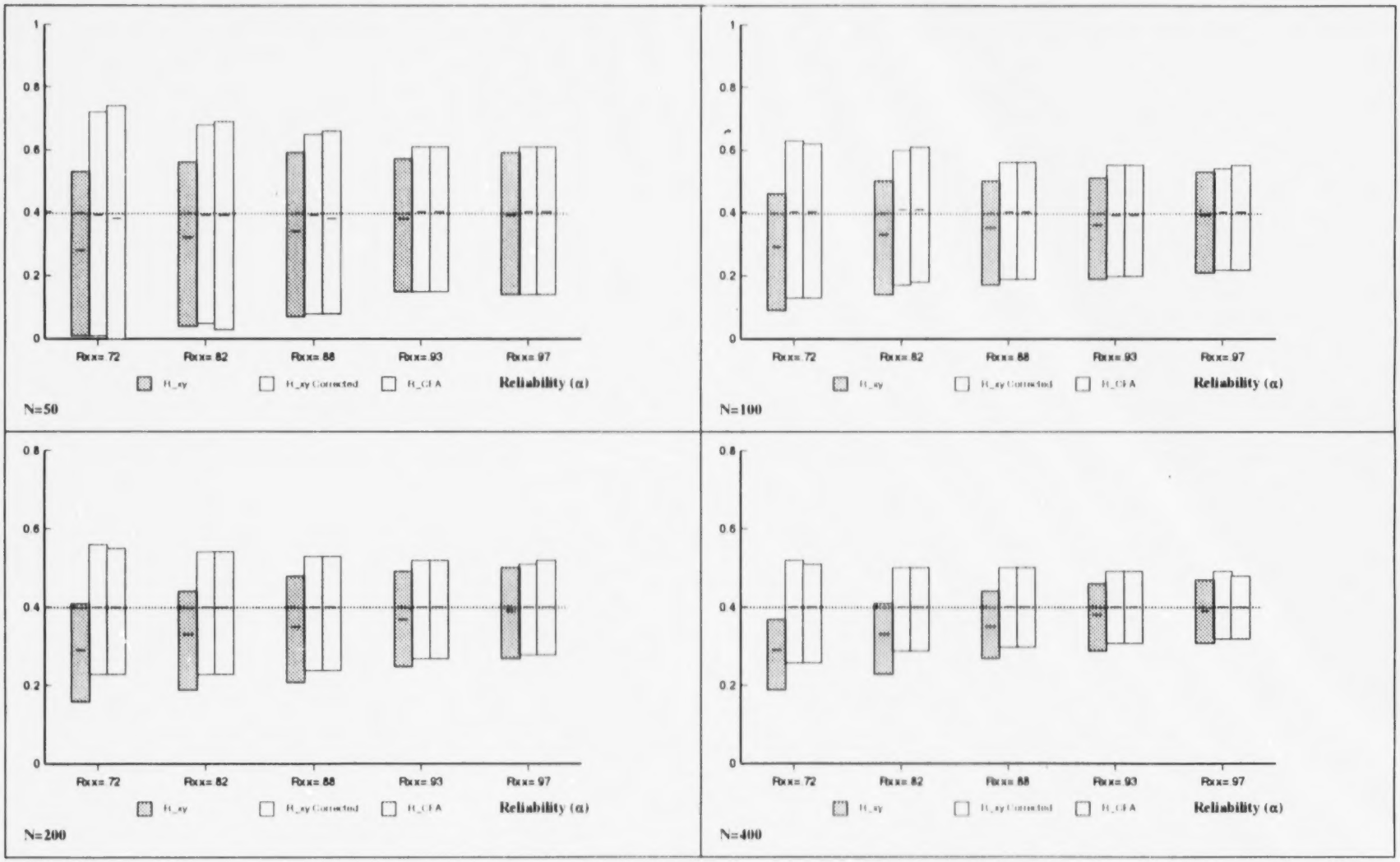

Figure 4 95% CI for Correlations between 8-Item Composites: Uncorrected (R_xy), Corrected for Unreliability (R_xy Corrected), and that from CFA Model (R_CFA) - Inter-Factor $\rho = 0.40$

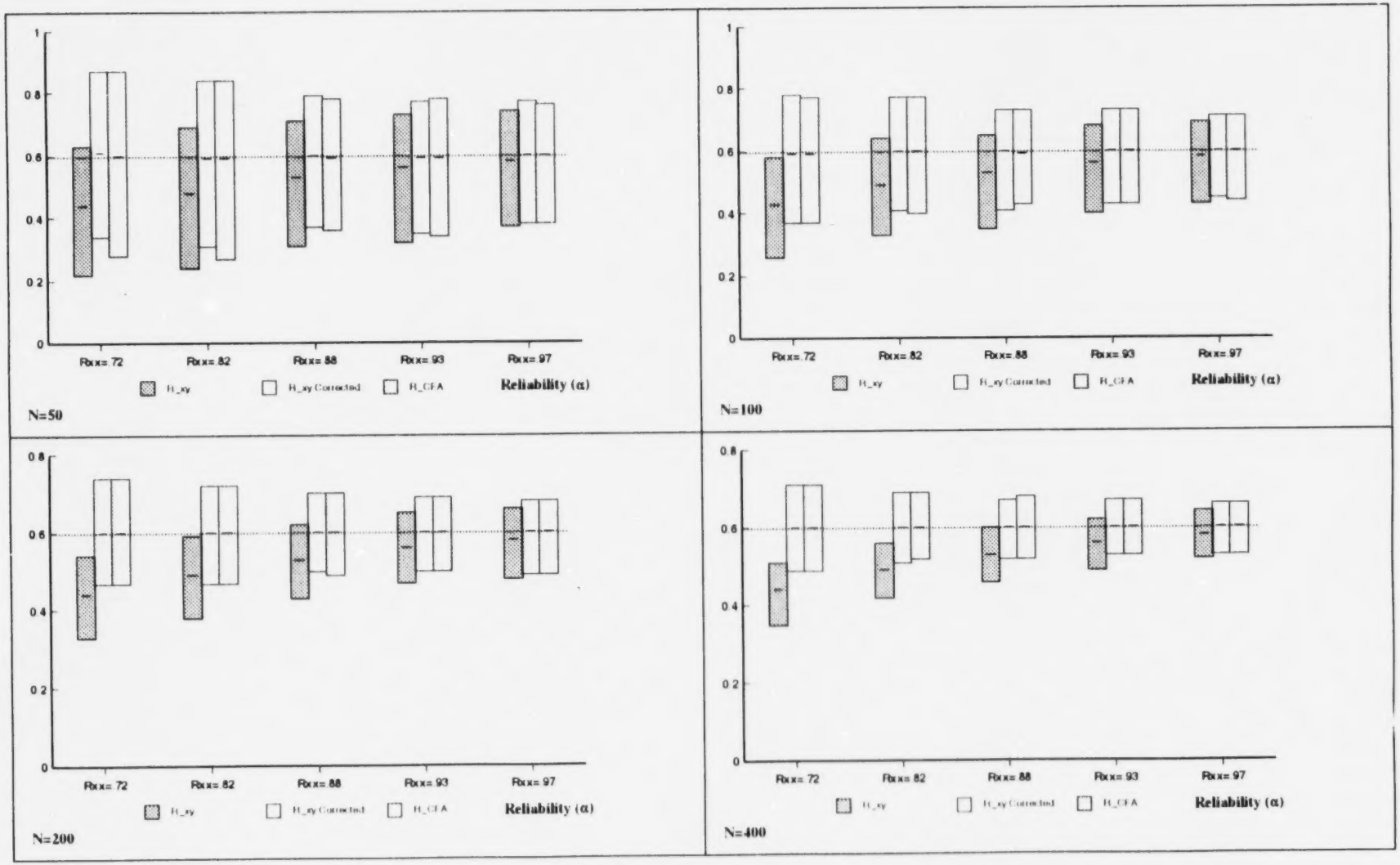

Figure 5 95% CI for Correlations between 8-Item Composites: Uncorrected (R_xy), Corrected for Unreliability (R_xy Corrected), and that from CFA Model (R_CFA) - Inter-Factor $\rho = 0.60$